Quadratic function and solving for x

1)

- Let be a quadratic function that intersects the x-axis at $x = 4$ and $x = 6$, and intersects the y-axis at $y = -24$. Find the formula for f and indicate its set of positivity.
- Find all x belonging to the real numbers that satisfy $f(x) = x - 6$.

2)

- Solve for x: $3^{\sqrt{12x^2-8}} = 9^x$
- Solve for x: $log_{x-2}64 = 2$

Answers

1)

- As the function intersects the x-axis at those two values, we know that those values are the roots, therefore we use the factored expression of the quadratic function: $y = a(x-4)(x-6)$. To find the value of a, we will use the fact that the other point is $(0, -24)$ since intersecting the y-axis means that the function passes through $x = 0$. Therefore, we have:

$$-24 = a(0-4)(0-6)$$
$$-24 = a(-4)(-6)$$
$$-24 = a(24)$$
$$-1 = a$$

The final result would be:

$$y = -1(x-4)(x-6)$$

To find the positivity set, we can do the following:

$$-1(x-4)(x-6) > 0$$
$$-(x^2 - x6 - x4 + 24) > 0$$
$$-x^2 + 10x - 24 > 0$$

Knowing that the roots of this function are 4 and 6, we substitute a value less than 4 into this function and get a negative value for y. We substitute a value between 4 and 6 into this function and get a positive value for y. Lastly, we substitute a value greater than 6 into this function and get another negative value for y. The positivity set is $(4, 6)$. We could also obtain this by graphing the function.

- We set up the equation:

$$-1(x-4)(x-6) = x-6$$
$$-x^2 + 10x - 24 = x - 6$$
$$-x^2 + 9x + 18 = 0$$

We obtain the values of the roots using the quadratic formula: $x = 3$ and $x = 6$. To verify, we substitute these values of x into both functions ($f(x)$ and $x - 6$) and we should get the same values.

2)

- To lower the exponents, we apply logarithms to both sides. Since we have 3 and 9, we can apply logarithm base 3:

$$log_3(3^{\sqrt{12x^2-8}}) = log_3(9^x)$$
$$\sqrt{12x^2-8}log_3(3) = xlog_3(9)$$
$$\sqrt{12x^2-8} = x2$$

We square both sides (ignoring the modulus for the moment):

$$12x^2 - 8 = (x^2 2^2)$$
$$12x^2 - 8 - 4x^2 = 0$$
$$8x^2 - 8 = 0$$

This leads us to $x = 1$. Note that although another root is $x = -1$, when we square both sides, we should have applied the modulus, so we have to take into account that one of the solutions may be wrong. To check this, we always replace the obtained roots in both equations and see if the equality holds. With $x = -1$, the equality at the beginning is not satisfied.

- The idea here is to use the following property: $log_b a = c$ then $b^c = a$

$$(x-1)^2 = 81$$

We raise both sides to the square root:

$$x - 1 = \sqrt{81}$$

or

$$x - 1 = -\sqrt{81}$$

The existence of these two possible results has to do with the fact that since $(x-1)^2$ is squared, the number inside could be positive or negative and still end up being a positive number (any number squared results in a positive number).

$$x = 10$$

$$x = -8$$

Once we have obtained these two results, we check them by inserting the values of x into the two expressions that should be equal. We obtain that the expressions are only equal with $x = 10$ and not with $x = -8$.